1997 AP Calculus BC: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_0^1 \sqrt{x}\left(x+1\right)dx =$

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

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2. If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\frac{e^{2t}}{\cos(2t)}$ (C) $\frac{\sin(2t)}{2e^{2t}}$ (D) $\frac{\cos(2t)}{2e^{2t}}$ (E) $\frac{\cos(2t)}{e^{2t}}$

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3. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) -1 (B) $-\frac{\sqrt{5}}{5}$ (C) 0 (D) $\frac{\sqrt{5}}{5}$ (E) 1

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4. $\frac{d}{dx}\left(xe^{\ln x^2}\right) =$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

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5. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

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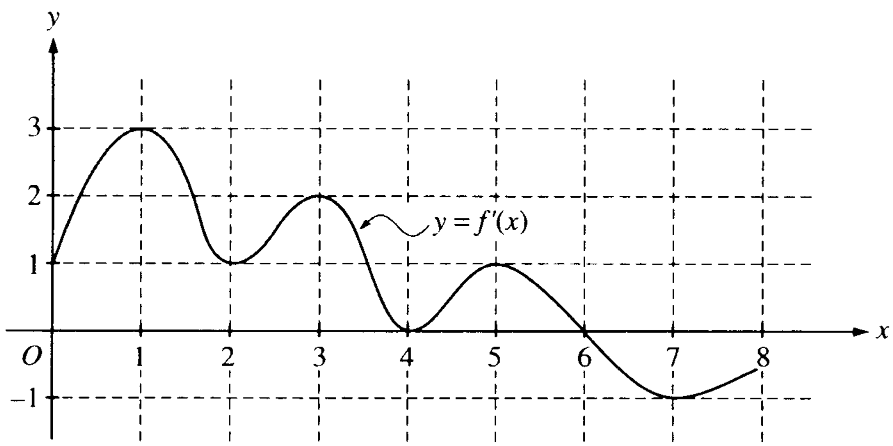
6. The line normal to the curve $y = \sqrt{16-x}$ at the point $\left(0,4\right)$ has slope

(A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) -8

Questions 7-9 refer to the graph and the information below.



The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

7. The point $(3,5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3,5)$ is

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x - 3)$

(D) $y + 5 = 2(x - 3)$

(E) $y + 5 = 2(x + 3)$

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8. How many points of inflection does the graph of f have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

9. At what value of x does the absolute minimum of f occur?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

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10. If $y = xy + x^2 + 1$, then when $x = -1$, $\dfrac{dy}{dx}$ is

(A) $\dfrac{1}{2}$ (B) $-\dfrac{1}{2}$ (C) -1 (D) -2 (E) nonexistent

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11. $\displaystyle\int_1^{\infty} \frac{x}{(1+x^2)^2}\,dx$ is

(A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{4}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2}$ (E) divergent

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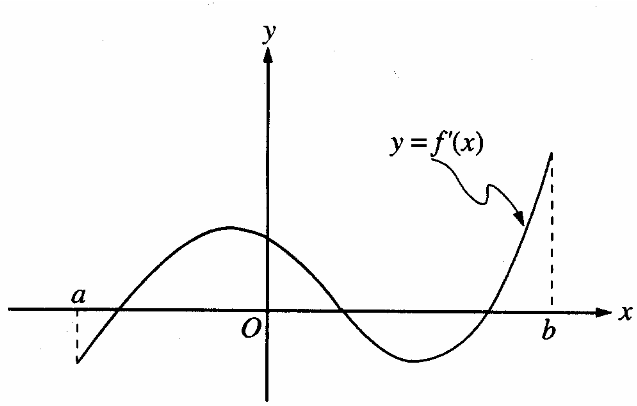

12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

13. A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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14. The sum of the infinite geometric series $\frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1{,}024} + \ldots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

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15. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t}\, dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t}\, dt$

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16. $\lim\limits_{h \to 0} \frac{e^h - 1}{2h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) e (E) nonexistent

17. Let f be the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

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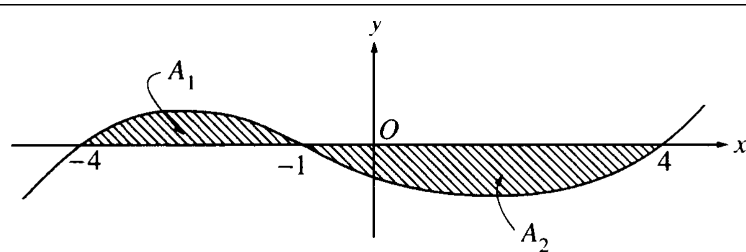
18. For what values of t does the curve given by the parametric equations $x=t^3-t^2-1$ and $y=t^4+2t^2-8t$ have a vertical tangent?

(A) 0 only
(B) 1 only
(C) 0 and $\frac{2}{3}$ only
(D) 0, $\frac{2}{3}$, and 1
(E) No value



19. The graph of $y=f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$\int_{-4}^{4} f(x)\,dx-2\int_{-1}^{4} f(x)\,dx=$

(A) A_1 (B) A_1-A_2 (C) $2A_1-A_2$ (D) A_1+A_2 (E) A_1+2A_2

20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ converges?

(A) $-3 \le x \le 3$
(B) $-3 < x < 3$
(C) $-1 < x \le 5$
(D) $-1 \le x \le 5$
(E) $-1 \le x < 5$

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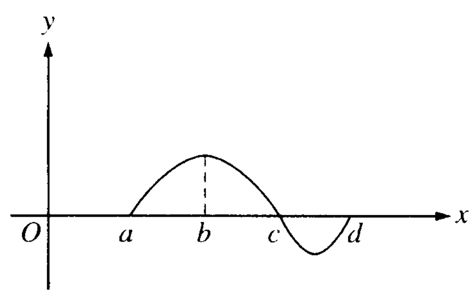
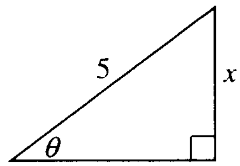
21. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$ (B) $3\int_0^{\pi} \cos^2\theta \, d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta \, d\theta$ (E) $3\int_0^{\pi} \cos\theta \, d\theta$



22. The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.



23. In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

---

24. The Taylor series for $\sin x$ about $x = 0$ is $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$. If f is a function such that $f'(x) = \sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

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25. The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x$?

(A) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$

(C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(D) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$

(E) $2\left(b^{\frac{1}{2}} - a^{\frac{1}{2}}\right)$

40 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. Which of the following sequences converge?

I. $\left\{\dfrac{5n}{2n-1}\right\}$

II. $\left\{\dfrac{e^n}{n}\right\}$

III. $\left\{\dfrac{e^n}{1+e^n}\right\}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

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77. When the region enclosed by the graphs of $y = x$ and $y = 4x - x^2$ is revolved about the y-axis, the volume of the solid generated is given by

(A) $\pi\int_0^3 \left(x^3 - 3x^2\right) dx$

(B) $\pi\int_0^3 \left(x^2 - \left(4x - x^2\right)^2\right) dx$

(C) $\pi\int_0^3 \left(3x - x^2\right)^2 dx$

(D) $2\pi\int_0^3 \left(x^3 - 3x^2\right) dx$

(E) $2\pi\int_0^3 \left(3x^2 - x^3\right) dx$

78. $\lim_{h\to 0} \dfrac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x) = \ln x$

(B) $f'(e)$, where $f(x) = \dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x) = \ln x$

(D) $f'(1)$, where $f(x) = \ln(x+e)$

(E) $f'(0)$, where $f(x) = \ln x$

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79. The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(A) Zero
(B) Three
(C) Five
(D) Six
(E) Seven

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80. Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

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81. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \dfrac{4}{9}$ for at least one c between –3 and 6

(C) $-1 \le f(x) \le 3$ for all x between –3 and 6

(D) $f(c) = 1$ for at least one c between –3 and 6

(E) $f(c) = 0$ for at least one c between –1 and 3

82. If $0 \le x \le 4$, of the following, which is the greatest value of x such that $\int_0^x (t^2 - 2t)\,dt \ge \int_2^x t\,dt$?

(A) 1.35 (B) 1.38 (C) 1.41 (D) 1.48 (E) 1.59

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83. If $\frac{dy}{dx} = (1 + \ln x)\,y$ and if $y = 1$ when $x = 1$, then $y =$

(A) $e^{\frac{x^2-1}{x^2}}$
(B) $1 + \ln x$
(C) $\ln x$
(D) $e^{2x + x\ln x - 2}$
(E) $e^{x \ln x}$

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84. $\int x^2 \sin x\,dx =$

(A) $-x^2 \cos x - 2x \sin x - 2\cos x + C$
(B) $-x^2 \cos x + 2x \sin x - 2\cos x + C$
(C) $-x^2 \cos x + 2x \sin x + 2\cos x + C$
(D) $-\frac{x^3}{3} \cos x + C$
(E) $2x \cos x + C$

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85. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

(A) None
(B) I only
(C) III only
(D) I and III only
(E) II and III only

86. $\int \frac{dx}{(x-1)(x+3)} =$

(A) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

(B) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

(E) $\ln\left|(x-1)(x+3)\right| + C$

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87. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\, dy$

(B) $\int_0^2 (2-y)\, dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$

88. Let $f(x)=\int_0^{x^2}\sin t\,dt$. At how many points in the closed interval $\left[0,\sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

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89. If f is the antiderivative of $\dfrac{x^2}{1+x^5}$ such that $f(1)=0$, then $f(4)=$

(A) -0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629

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90. A force of 10 pounds is required to stretch a spring 4 inches beyond its natural length. Assuming Hooke's law applies, how much work is done in stretching the spring from its natural length to 6 inches beyond its natural length?

(A) 60.0 inch-pounds
(B) 45.0 inch-pounds
(C) 40.0 inch-pounds
(D) 15.0 inch-pounds
(E) 7.2 inch-pounds

1997 Answer Key

1997 AB

1. C	21. E		
2. A	22. D		
3. C	23. A		
4. D	24. B		
5. E	25. A		
6. C	76. E		
7. D	77. D		
8. C	78. D		
9. B	79. C		
10. E	80. A		
11. E	81. A		
12. B	82. B		
13. A	83. C		
14. C	84. C		
15. B	85. C		
16. D	86. A		
17. A	87. B		
18. C	88. E		
19. D	89. B		
20. E	90. D		

1997 BC

1. C	21. A
2. E	22. C
3. A	23. E
4. C	24. D
5. C	25. A
6. A	76. D
7. C	77. E
8. E	78. A
9. A	79. D
10. B	80. B
11. C	81. D
12. A	82. B
13. B	83. E
14. C	84. C
15. D	85. D
16. B	86. A
17. B	87. B
18. C	88. C
19. D	89. D
20. E	90. B

1997 Calculus BC Solutions: Part A

1. C $\int_0^1 \sqrt{x}(x+1)\,dx = \int_0^1 x^{\frac{3}{2}} + x^{\frac{1}{2}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{16}{15}$

2. E $x = e^{2t}$, $y = \sin(2t)$; $\frac{dy}{dx} = \frac{2\cos(2t)}{2e^{2t}} = \frac{\cos(2t)}{e^{2t}}$

3. A $f(x) = 3x^5 - 4x^3 - 3x$; $f'(x) = 15x^4 - 12x^2 - 3 = 3(5x^2+1)(x^2-1) = 3(5x^2+1)(x+1)(x-1)$; f' changes from positive to negative only at $x = -1$.

4. C $e^{\ln x^2} = x^2$; so $xe^{\ln x^2} = x^3$ and $\frac{d}{dx}(x^3) = 3x^2$

5. C $f(x) = (x-1)^{\frac{3}{2}} + \frac{1}{2}e^{x-2}$; $f'(x) = \frac{3}{2}(x-1)^{\frac{1}{2}} + \frac{1}{2}e^{x-2}$; $f'(2) = \frac{3}{2} + \frac{1}{2} = 2$

6. A $y = (16-x)^{\frac{1}{2}}$; $y' = -\frac{1}{2}(16-x)^{-\frac{1}{2}}$; $y'(0) = -\frac{1}{8}$; The slope of the normal line is 8.

7. C The slope at $x = 3$ is 2. The equation of the tangent line is $y - 5 = 2(x-3)$.

8. E Points of inflection occur where f' changes from increasing to decreasing, or from decreasing to increasing. There are six such points.

9. A f increases for $0 \le x \le 6$ and decreases for $6 \le x \le 8$. By comparing areas it is clear that f increases more than it decreases, so the absolute minimum must occur at the left endpoint, $x = 0$.

10. B $y = xy + x^2 + 1$; $y' = xy' + y + 2x$; at $x = -1, y = 1$; $y' = -y' + 1 - 2 \Rightarrow y' = -\frac{1}{2}$

11. C $\int_1^\infty x(1+x^2)^{-2}\,dx = \lim_{L\to\infty} -\frac{1}{2}(1+x^2)^{-1}\Big|_1^L = \lim_{L\to\infty} \frac{1}{4} - \frac{1}{2(1+L^2)} = \frac{1}{4}$

12. A f' changes from positive to negative once and from negative to positive twice. Thus one relative maximum and two relative minimums.

13. B $a(t) = 2t - 7$ and $v(0) = 6$; so $v(t) = t^2 - 7t + 6 = (t-1)(t-6)$. Movement is right then left with the particle changing direction at $t = 1$, , therefore it will be farthest to the right at $t = 1$.

14. C Geometric Series. $r = \frac{3}{8} < 1 \Rightarrow$ convergence. $a = \frac{3}{2}$ so the sum will be $S = \dfrac{\frac{3}{2}}{1-\frac{3}{8}} = 2.4$

15. D $x = \cos^3 t,\ y = \sin^3 t$ for $0 \le t \le \frac{\pi}{2}$. $L = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$L = \int_0^{\pi/2} \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\, dt = \int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$$

16. B $\lim\limits_{h\to 0} \frac{e^h - 1}{2h} = \frac{1}{2} \lim\limits_{h\to 0} \frac{e^h - e^0}{h} = \frac{1}{2} f'(0)$, where $f(x) = e^x$ and $f'(0) = 1$. $\lim\limits_{h\to 0} \frac{e^h - 1}{2h} = \frac{1}{2}$

17. B $f(x) = \ln(3-x);\ f'(x) = \frac{1}{x-3},\ f''(x) = -\frac{1}{(x-3)^2},\ f'''(x) = \frac{2}{(x-3)^3};$

$f(2) = 0,\ f'(2) = -1,\ f''(2) = -1,\ f'''(2) = -2;\ \ a_0 = 0, a_1 = -1, a_2 = -\frac{1}{2}, a_3 = -\frac{1}{3}$

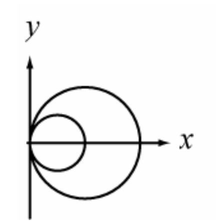
$f(x) \approx -(x-2) - \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$

18. C $x = t^3 - t^2 - 1,\ y = t^4 + 2t^2 - 8t;\ \ \frac{dy}{dx} = \frac{4t^3 + 4t - 8}{3t^2 - 2t} = \frac{4t^3 + 4t - 8}{t(3t-2)}$. Vertical tangents at $t = 0, \frac{2}{3}$

19. D $\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx = (A_1 - A_2) - 2(-A_2) = A_1 + A_2$

20. E $\sum\limits_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$. The endpoints of the interval of convergence are when $(x-2) = \pm 3;\ x = -1, 5$.
Check endpoints: $x = -1$ gives the alternating harmonic series which converges. $x = 5$ gives the harmonic series which diverges. Therefore the interval is $-1 \le x < 5$.

21. A Area $= 2 \cdot \frac{1}{2} \int_0^{\pi/2} ((2\cos\theta)^2 - \cos^2\theta)\, d\theta = \int_0^{\pi/2} 3\cos^2\theta\ d\theta$



22. C $g'(x) = f(x)$. The only critical value of g on (a, d) is at $x = c$. Since g' changes from positive to negative at $x = c$, the absolute maximum for g occurs at this relative maximum.

23. E $x = 5\sin\theta;\ \frac{dx}{dt} = 5\cos\theta \cdot \frac{d\theta}{dt};$ When $x = 3, \cos\theta = \frac{4}{5};\ \frac{dx}{dt} = 5\left(\frac{4}{5}\right)(3) = 12$

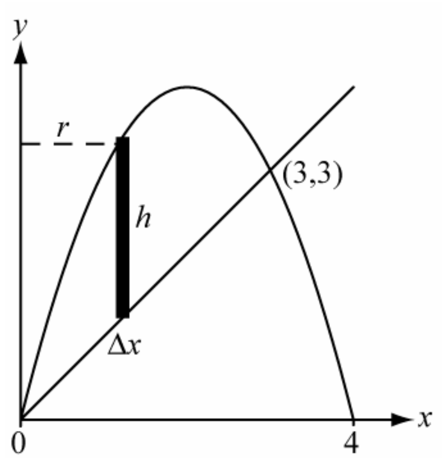
24. D $f'(x) = \sin(x^2) = x^2 - \frac{(x^2)^3}{3!} + \cdots = x^2 - \frac{1}{6}x^6 + \cdots \Rightarrow f(x) = \frac{1}{3}x^3 - \frac{1}{42}x^7 + \cdots$ The coefficieint of x^7 is $-\frac{1}{42}$.

25. A This is the limit of a right Riemann sum of the function $f(x) = \sqrt{x}$ on the interval $[a,b]$, so

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x = \int_a^b \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_a^b = \frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$$

76. D Sequence $\mathrm{I} \to \frac{5}{2}$; sequence $\mathrm{II} \to \infty$; sequence $\mathrm{III} \to 1$. Therefore I and III only.

77. E Use shells (which is no longer part of the AP Course Description.)



$\sum 2\pi r h \Delta x$ where $r = x$ and
$h = 4x - x^2 - x$

Volume =
$2\pi \int_0^3 x(4x - x^2 - x)\,dx = 2\pi \int_0^3 (3x^2 - x^3)\,dx$

78. A $\lim_{h \to 0} \frac{\ln(e+h) - 1}{h} = \lim_{h \to 0} \frac{\ln(e+h) - \ln e}{h} = f'(e)$ where $f(x) = \ln x$

79. D Count the number of places where the graph of $y(t)$ has a horizontal tangent line. Six places.

80 B Find the first turning point on the graph of $y = f'(x)$. Occurs at $x = 0.93$.

81. D f assumes every value between -1 and 3 on the interval $(-3, 6)$. Thus $f(c) = 1$ at least once.

82. B $\int_0^x (t^2 - 2t)\,dt \ge \int_2^x t\,dt$; $\frac{1}{3}x^3 - x^2 \ge \frac{1}{2}x^2 - 2$. Using the calculator, the greatest x value on the interval $[0, 4]$ that satisfies this inequality is found to occur at $x = 1.3887$.

83. E $\frac{dy}{y} = (1 + \ln x)\,dx$; $\ln|y| = x + x\ln x - x + k = x\ln x + k$; $|y| = e^k e^{x\ln x} \Rightarrow y = Ce^{x\ln x}$. Since $y = 1$ when $x = 1$, $C = 1$. Hence $y = e^{x\ln x}$.

84. C $\int x^2 \sin x\,dx$; Use the technique of antiderivatives by parts with $u = x^2$ and $dv = \sin x\,dx$. It will take 2 iterations with a different choice of u and dv for the second iteration.

$$\begin{aligned}\int x^2 \sin x\,dx &= -x^2 \cos x + \int 2x \cos x\,dx \\ &= -x^2 \cos x + \left(2x \sin x - \int 2 \sin x\,dx\right) \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + C\end{aligned}$$

85. D I. Average rate of change of f is $\dfrac{f(3) - f(1)}{3-1} = \dfrac{5}{2}$. True

II. Not enough information to determine the average value of f. False

III. Average value of f' is the average rate of change of f. True

86. A Use partial fractions. $\dfrac{1}{(x-1)(x+3)} = \dfrac{A}{x-1} + \dfrac{B}{x+3}$; $1 = A(x+3) + B(x-1)$

Choose $x = 1 \Rightarrow A = \dfrac{1}{4}$ and choose $x = -3 \Rightarrow B = -\dfrac{1}{4}$.

$$\int \frac{1}{(x-1)(x+3)}\,dx = \frac{1}{4}\left[\int \frac{1}{x-1}\,dx - \int \frac{1}{x+3}\,dx\right] = \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$$

87. B Squares with sides of length x. Volume = $\int_0^2 x^2 dy = \int_0^2 (2-y)\,dy$

88. C $f(x) = \int_0^{x^2} \sin t\,dt$; $f'(x) = 2x\sin(x^2)$; For the average rate of change of f we need to determine $f(0)$ and $f(\sqrt{\pi})$. $f(0) = 0$ and $f\left(\sqrt{\pi}\right) = \int_0^{\pi} \sin t\,dt = 2$. The average rate of change of f on the interval is $\dfrac{2}{\sqrt{\pi}}$. See how many points of intersection there are for the graphs of $y = 2x\sin(x^2)$ and $y = \dfrac{2}{\sqrt{\pi}}$ on the interval $\left[0, \sqrt{\pi}\right]$. There are two.

89. D $f(x)=\int_1^x \frac{t^2}{1+t^5}\,dt;\ f(4)=\int_1^4 \frac{t^2}{1+t^5}\,dt=0.376$

Or, $f(4)=f(1)+\int_1^4 \frac{x^2}{1+x^5}\,dx=0.376$

Both statements follow from the Fundamental Theorem of Calculus.

90. B $F(x)=kx;\ 10=4k\ \Rightarrow\ k=\frac{5}{2};$ Work $=\int_0^6 F(x)\,dx=\int_0^6 \frac{5}{2}x\,dx=\frac{5}{4}x^2\Big|_0^6=45$ inch-lbs